# GRAPH-PROJECTED SIGNAL PROCESSING

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## 1. Context

- **Graph Signal Processing (GSP)** [5,7] is a promising framework to extend Fourier analysis to complex topological domains described using graphs,
- It builds upon **analogies** between ring **graphs** spectrum and classical **1D Fourier transform**,
- This analogy is **questioned** as soon as we consider **2D graphs** as the obtained transforms differ from usual **2D** Fourier transforms,
- We propose an **alternative** to define **signal processing on graphs**, which aims to embed vertices into $d$-dimensional integer coordinates while preserving distances between pairs of vertices,
- We perform **experiments** on vision datasets and obtain significant gains in performance when compared to regular GSP-inspired CNNs.

## 2. Methodology

1. Start with a (possibly weighted) graph $G = \langle V, E \rangle$,
2. Find a *reasonable* injective embedding
$$\varphi : \begin{cases} V \to \mathbb{Z}^d \\ v \mapsto \varphi(v) \end{cases},$$
3. Define signal processing tools on $G$ by considering the usual $d$-dimensional such transforms on $\mathbb{Z}^d$ mapped to $V$ using $\varphi^{-1}$.

## 3. Reasonable embedding

We propose the following embedding, called *optimal embedding*:
$$\varphi_\alpha = \arg\min_{\varphi} c_\alpha(\varphi),$$
with
$$c_\alpha(\varphi) = \sum_{v,v' \in V} |\alpha \|\varphi(v) - \varphi(v')\|_1 - d_G(v,v')|,$$
where $d_G(v,v')$ denotes the distance between $v$ and $v'$ in $G$ (we choose geodesic in our experiments, but could be resistive or any distance on $G$) and $\alpha$ is a scaling factor. Other embeddings have been proposed in $\mathbb{R}^2$ in the literature [3].

## 4. Optimization algorithm

We propose a mix of gradient descent and a barrier method:

**Inputs:** Graph $G$, dimension $d$, scale $\alpha$, parameters $\delta, \beta$.

1) $\varphi \leftarrow$ random embedding;

2) $$\varphi \leftarrow GradMin_{\varphi'}\left(\frac{c_\alpha(\varphi') + \beta_i \left[\sum_v d_1(\varphi'(v), \mathbb{Z}^d)\right]}{1+\beta_i}, \varphi\right);$$

3) $\beta \leftarrow \delta\beta$;

Repeat from 2.

We start by focusing only on minimizing the cost $c_\alpha(\varphi)$, and we increase at each step the constraint of forcing the embedding to be in $\mathbb{Z}^d$.

## 5. Embeddings

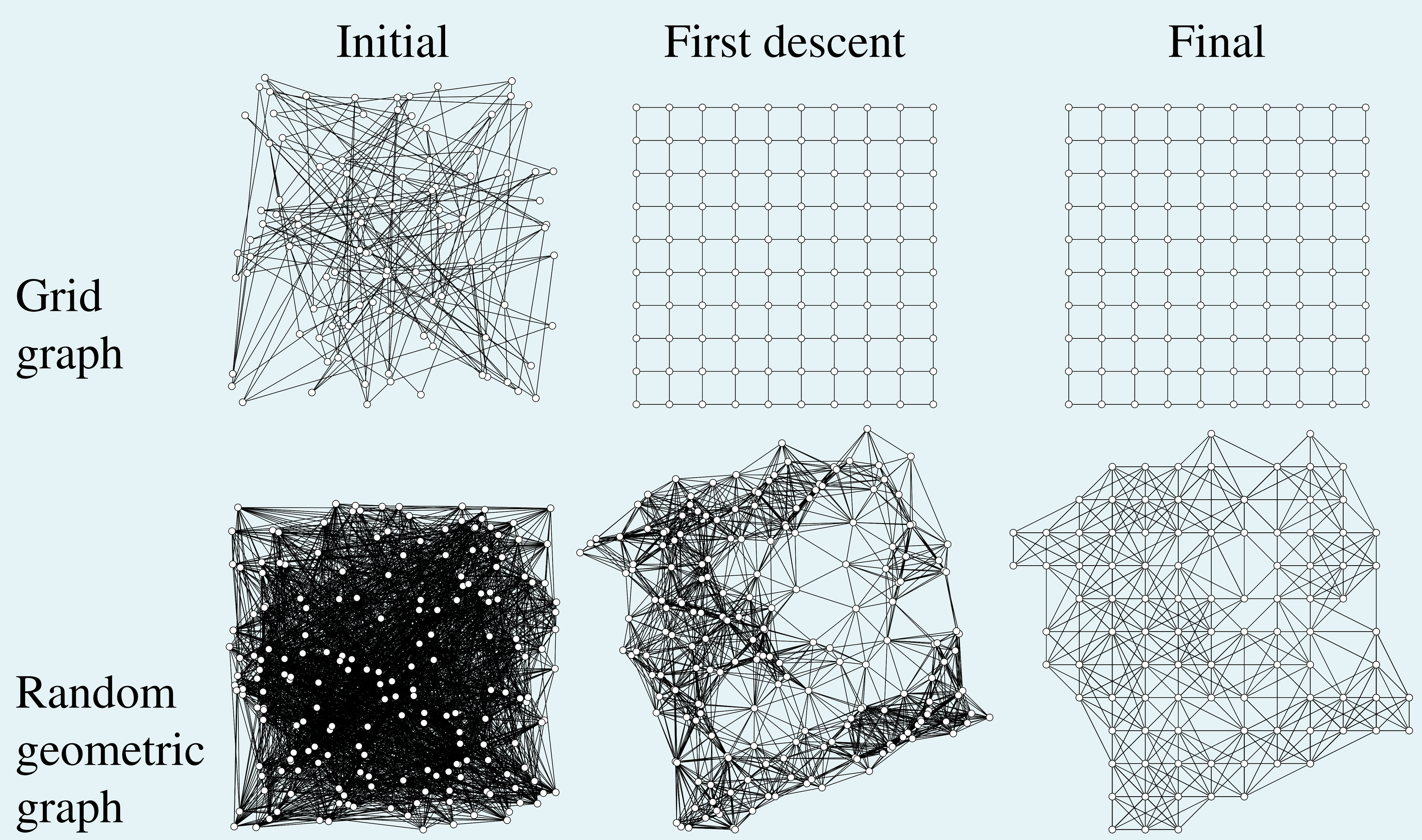


Figure: Illustration of successive steps in the proposed optimisation procedure. Left column is initial random embedding, middle column is after the first gradient descent, and right column at the end of the process.

## 6. Fourier modes

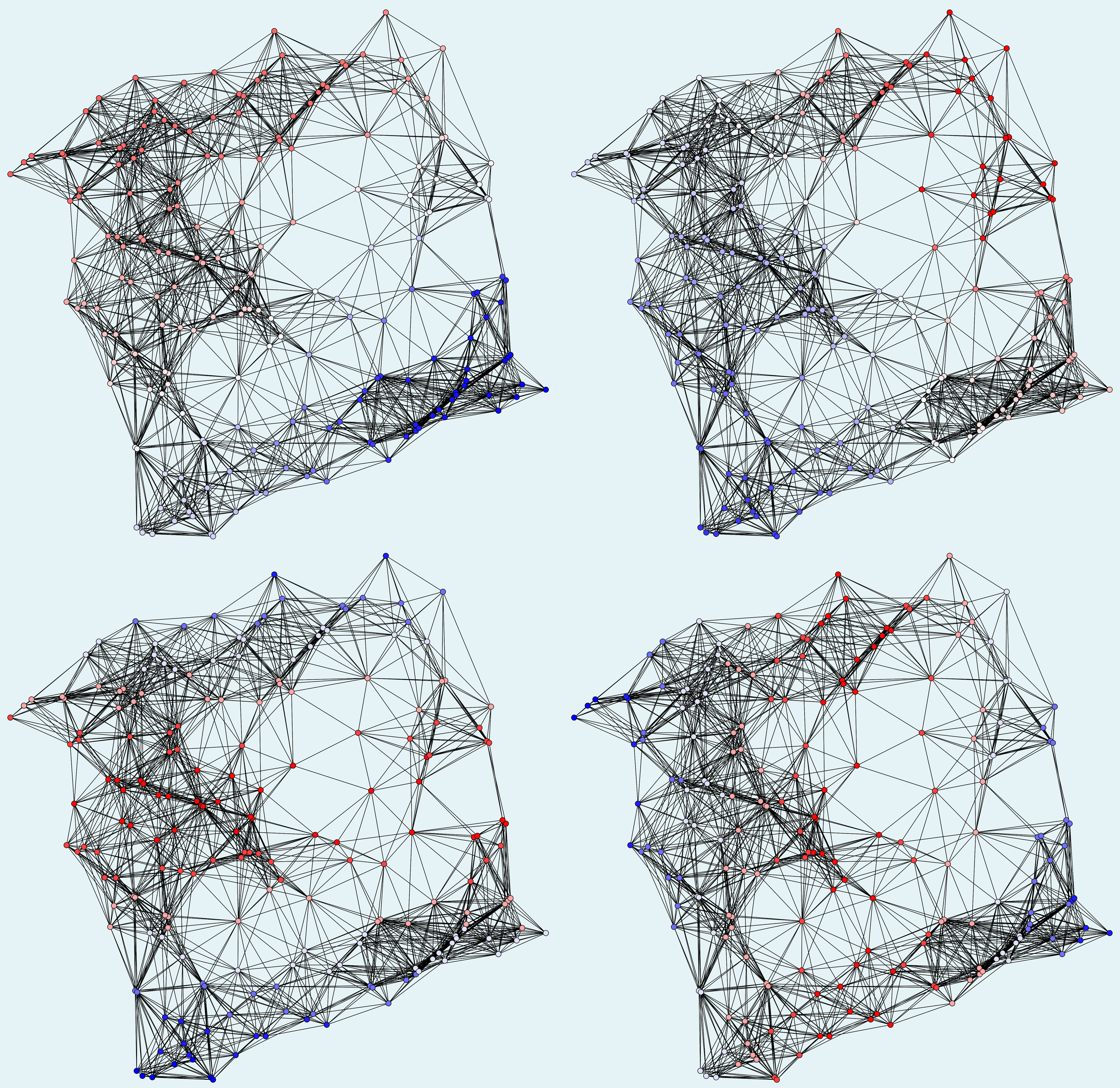

Figure: Depiction of the first eigenvectors/Fourier modes (associated with the lowest nonzero eigenvalues/frequencies) of a random geometric graph. First line corresponds to the classical graph signal processing definition (using the Laplacian), and second line to the proposed embedding.

## 6. Experiments

We compare our method against CNNs and graph CNNs in regular 2D grids (CIFAR-10) and irregular 3D grid (PINES) scenarios:

CIFAR-10 test set accuracy comparison table.

| | | | Classical GSP | Others | |
|---|---|---|---|---|---|
| MLP [4] | CNN [8] | Support | [2] | [6] | **Proposed** |
| 78.62% | 92.73% | Grid | 84.41% | 92.81% | 92.73% |
| | | Covariance | —— | 91.07% | 89.25% |

PINES [9] test set accuracy comparison table.

| Graph | None | | Neighborhood Graph | | |
|---|---|---|---|---|---|
| Method | Dense | CNN | [2] | [6] | **Proposed** |
| Accuracy | 82.62% | 85.47% | 82.80% | 85.08% | 84.78% |

## 7. Conclusion

- We proposed an **alternative** to design **signal processing on graphs**,
- Our method uses an **optimization routine** to find an embedding of a graph into a regular $d$-dimensional discrete domain,
- The proposed method **outperforms classical GSP-inspired CNNs**.

## 8. Future work

- We would like to **better motivate** the choice of the optimal embedding,
- We want to **stress** our method using **other applications of GSP**,
- A more thorough **theoretical analysis** is required.

## 9. References

[1] "Geometric deep learning: going beyond euclidean data," 2017.
[2] "Convolutional neural networks on graphs with fast localized spectral filtering," 2016.
[3] "Force-directed drawing algorithms", 2004.
[4] "How far can we go without convolution: Improving fully-connected networks," 2015.
[5] "Graph signal processing," 2017.
[6] "Matching Convolutional Neural Networks without Priors about Data," 2018.
[7] "The emerging field of signal processing on graphs: Extending high-dimensional data analysis to networks and other irregular domains," 2013.
[8] "Identity mappings in deep residual networks," 2016.
[9] "A sensitive and specific neural signature for picture-induced negative affect," 2015.

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